Extraction cost: before or after harvesting? Economic and environmental consequences

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Context of the paper

- Groundwater extraction: marginal cost depends on the level of the aquifer
- In general, resources with accessibility problems: cost depends on scarcity
- The main ingredient: make the cost depend on the projected evolution of the resource: before or after the extraction or rainfall
- The goal: deduce its economical and environmental consequences
- The method: revisit the discrete time, infinite horizon dynamic game model of Provencher and Burt (JEEM 1993).

Contributions

We analyze a variant of Provencher and Burt's model, in the linear-quadratic case.

- We characterize the existence of Nash equilibria in affine feedback
- We prove qualitative properties of equilibria as a function of the discount factor.
- We discuss the particular cases: myopic (zero discount factor) and "golden rule" (discount factor tending to one).
- We focus on the case of scarse resources and find that taking harvesting and rainfall into account in the cost is better than the standard situation.

Criteria of evaluation

Criteria for defining the "better" situation

- individual welfare of players
- state of the resource
    - asymptotic (steady state)
    - transient (not in this study)
- social welfare (not in this study)

The model of Provencher and Burt

We consider the extraction of groundwater by two players.
The dynamic of groundwater:

$$G_{t+1} = G_t + R - u_t^1 - u_t^2, \quad G_0, \quad \text{given}.$$

We suppose R is a constant.

The instantaneous profit:

$$\pi_i = F_i(u_t^i) - C_i(G_t) \ \times \ u_t^i.$$

The marginal extraction cost ($C_i(.)$) depends on the current level of the groundwater.

The extended model

Introduce the more general instantaneous profit function:

$$\pi_i(u_t^i) = F_i(u_t^i) - C_i(G_t + mR - n(u_t^1 + u_t^2))\ u_t^i$$

where $n, m \in [0, 1]$.
The extreme cases:

- $n = 0, m = 0$ (the standard case): cost based on current resource
- $n = 1, m = 1$: cost based on the state of the resource in the following period.

When $n \neq 0$ the profit function of player i depends on the action of the other player: strategic interaction not just through the dynamics.

The dynamic game setting

We formulate a discrete time, infinite-horizon, discounted dynamic game: for player $i \in \{1, 2\}$,

$$\max_{\{u_t^i\}_t} \sum_0^\infty \beta^t [F_i(u_t^i) - C_i(G_t + mR - n(u_t^1 + u_t^2))\ u_t^i],$$

such that

$$G_{t+1} = G_t + R - u_t^1 - u_t^2, \quad G_0, \quad \text{given.}$$

The Bellman equation associated with a Nash Feedback equilibrium is:

$$\begin{aligned} V^i(G_0) \quad = \quad & \max_{u_t^i} \Big[F_i(u^i) - C_i(G + mR - n(u_t^1 + u_t^2))u^i \\ & + \beta V^i(G_t + R - (u_t^1 + u_t^2))\Big]. \end{aligned}$$

Analysis of the model

- solution in the linear-quadratic case
- exercise in sensitivity analysis
- still work in progress!

Solution in the Linear-Quadratic case

Assume:

$$F_i(u) = u - \frac{b_i}{2}u^2, \quad C_i(x) = z_i - c_i x > 0.$$

We propose as solution of the Bellman equation:

- a value function for Player $i \in \{1,2\}$ of the form

$$V^i(G) = \frac{A_i}{2}G^2 + B_i G + C_i,$$

- a feedback law of the form

$$u^i = \alpha_i G + \gamma_i.$$

The unknown A_i, B_i, C_i, α_i, γ_i, $i = 1,2$, are found identifying

- the coefficients of the quadratic function in the Bellman equation after optimization
- the optimal strategy for j and the one conjectured by i.

Solution in the Linear-Quadratic case (ctd)

More precisely, a solution by stages:

- system of 3rd degree polynomial equations for $\{\alpha_1, \alpha_2\}$
- A_1 and A_2 as simple functions of $\{\alpha_1, \alpha_2\}$

$$A_i = \frac{c_i(1-\alpha_j n) - \alpha_i(b + 2c_i n)}{\beta(1-\alpha_1-\alpha_2)}$$

- linear system for $\{B_1, B_2, \gamma_1, \gamma_2\}$
- simple formulas for $\{C_1, C_2\}$.

Interior solutions

The existence of a useful solution is not granted because:

- the LQ problem is not concave

$$\pi^i(u) \;=\; u(1 - z_i - \frac{b_i}{2}u) + c_i u G$$

- there are physical constraints:
  positive harvesting (!)

$$u_t^i \geq 0, \qquad \forall t$$

  positive and bounded stock

$$\overline{G} \geq G_t \geq 0, \qquad \forall t.$$

$\Longrightarrow$ we find only interior solutions.

Equilibrium trajectories

When affine feedback controls $u^i = \alpha_i G + \gamma_i$ are implemented, the dymamics becomes:

$$G_t = (1 - \alpha_1 - \alpha_2) G_{t-1} + R - \gamma_1 - \gamma_2$$

with solution:

$$G_t = (1 - \alpha_1 - \alpha_2)^t G_0 + \frac{R - \gamma_1 - \gamma_2}{\alpha_1 + \alpha_2} \left(1 - (1 - \alpha_1 - \alpha_2)^t\right).$$

Necessary conditions for the trajectory to be valid are:

- $0 \leq \alpha_1 + \alpha_2 < 2$
- $\gamma_1 + \gamma_2 \leq R$.

Otherwise, an interior solution of our problem does not exit.

The symmetric linear quadratic case

Symmetric case: look for symmetric equilibria: $\alpha_1 = \alpha_2 = \alpha$.
With the procedure described above we find that α solves:

$$\begin{aligned} p(Z) \; := \; & \beta \left\{ 2(b+2cn)Z^3 - (3b+8cn)Z^2 + 2cZ \right\} \\ & +(1-\beta)\left\{-(b+3cn)Z + c\right\} \; = \; 0. \end{aligned}$$

Existence and uniqueness of solution

There is one unique root of $p(Z)$ in $(0,1)$ if

$$c < \frac{b}{1+\beta}.$$

If $c < b/2$, this root actually satisfies:

$$0 \leq \alpha \leq \frac{c}{b+3cn} < \frac{1}{2}.$$

Solution (end)

Once α is determined, we can compute

$$
\begin{aligned}
A &= \frac{c - \alpha\,(b + 3cn)}{\beta(1 - 2\alpha)} \\
\gamma &= \frac{\alpha}{c}\left[1 - z + mcR - \frac{R[c - \alpha(b + 3cn)]}{(1 - 2\alpha)(1 - \beta + \alpha\beta)}\right] \\
B &= \ldots \\
C &= \ldots
\end{aligned}
$$

Observations:

- α and A do not depend on z, R or m
- γ and B depend linearly on m and R

Qualitative analysis

We investigate the variation of the equilibrium (feedback parameters, value)

- when the discount factor β varies
- when the cost adjustment parameters n (harvest) and m (rainfall) vary.

Variation with respect to β

When agents are more shortsighted, they react more agressively and the environment suffers, whatever the cost structure.

Monotony in β

Under the assumption $c < b/2$, the function:

- $\beta \mapsto \alpha(\beta)$ is decreasing on $[0, 1]$
- $\beta \mapsto \gamma(\beta)$ is decreasing on $[0, 1]$ when it is positive
- $\beta \mapsto G_\infty(\beta)$ is increasing when γ is positive.

We conjecture that $\gamma(\cdot)$ is actually always decreasing.

The green golden rule, $\beta = 1$

Note that the limit when β goes to 1 can allow to select a solution for the static green golden rule

$$\max_{u^i} F_i(u^i) - C_i(.)u^i, \quad \text{such that} \quad u^1 + u^2 = R.$$

As it is well know, this is a game with coupled constraints and there exists an infinite number of solutions.

In the symmetric case, $\lim_{t\to\infty} u^i_t = R/2$ for $i = 1, 2$ and all β.
But in the asymmetric case this limit can allow to select one equilibrium.
Observe that $\lim_{\beta\uparrow 1} \alpha(1) \neq 0$.

Comparing the cost situations

Lexicon for the cost mechanism:

tag	n	m	
00	0	0	before rainfall and harvest
$H0$	1	0	after harvest but before rainfall
$0R$	0	1	before harvest but after rainfall
HR	1	1	after harvest *and* rainfall

Since the equilibrium reaction rate α and the leading coefficient A do not depend on R, nor on m:

$$\alpha_{0R} = \alpha_{00} \qquad \alpha_{H0} = \alpha_{HR}$$
$$A_{0R} = A_{00} \qquad A_{H0} = A_{HR}.$$

Benchmark situation: the myopic case $\beta = 0$

When $\beta = 0$, we find:

$$u(G) = \underbrace{\frac{c}{b + 3cn}}_{\alpha(0)} \; G + \underbrace{\frac{1 - z + cmR}{b + 3cn}}_{\gamma(0)} .$$

The value function of both players is:

$$\pi(G_0) \;=\; \frac{(cG_0 + 1 - z + cmR)^2}{(b + 3cn)^2} \; \frac{b + 2cn}{2} .$$

And the asymptotic stock is:

$$G_\infty \;=\; \frac{R - 2\gamma(0)}{2\alpha(0)} \;=\; \frac{R(b + 3cm - 2cn) - 2(1 - z)}{2c} .$$

Clearly:

$$\frac{\partial \alpha(0)}{\partial n} < 0, \quad \frac{\partial \pi(G_0)}{\partial n} < 0, \quad \frac{\partial \gamma(0)}{\partial m} > 0, \quad \frac{\partial \pi(G_0)}{\partial m} > 0.$$

Accordingly:

Ranking of controls, gains and steady states

- Controls are ordered as: $u_{H0}(G) < u_{00}(G)$ and

$$u_{H0}(G) < u_{HR}(G), \qquad u_{00}(G) < u_{0R}(G)$$

- Value functions are ordered as: $\pi_{H0}(G) < \pi_{00}(G)$ and

$$\pi_{H0}(G) < \pi_{HR}(G), \qquad \pi_{00}(G) < \pi_{0R}(G)$$

- Steady-state stocks are ordered as:

$$G_{H0}^{\infty} > G_{HR}^{\infty} > G_{00}^{\infty} > G_{0R}^{\infty}.$$

Rankings (ctd)

Under additional conditions: if R small enough (scarse resource), controls and values can be totally ordered.

$$u_{H0}(G) < u_{HR}(G) < u_{00}(G) < u_{0R}(G)$$

$$\pi_{H0}(G) < \pi_{HR}(G) < \pi_{00}(G) < \pi_{0R}(G)$$

$$G^\infty_{H0} > G^\infty_{HR} > G^\infty_{00} > G^\infty_{0R}.$$

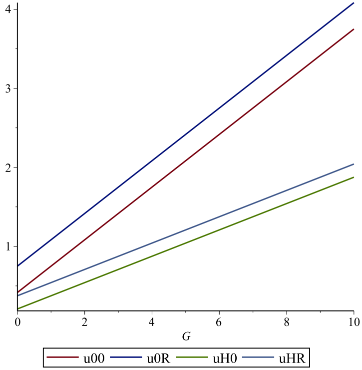
However, in order to have positive harvesting policy for all G and a positive steady state, we must impose

$$R > \frac{2(1-z)}{b-2c}$$

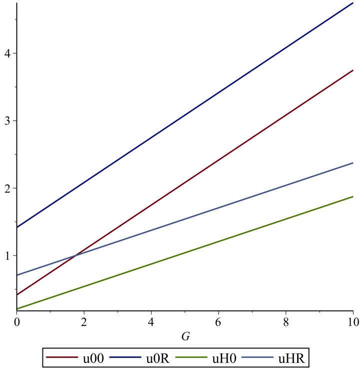
that is, R "not too small".

Numerical illustration

Case of interior feedback but different rankings for controls



R small



R large

Variation of α and γ

Towards a generalization of these findings to general $\beta \in [0, 1]$.

Variations of α and γ

- The function $n \mapsto \alpha(n)$ is decreasing, so that,

$$\alpha_{0R} = \alpha_{00} \;>\; \alpha_{H0} = \alpha_{HR}.$$

- The function $m \mapsto \gamma(m)$ is increasing, so that:

$$\gamma_{H0} < \gamma_{HR}, \quad \gamma_{00} < \gamma_{0R}$$

and consequently:

$$u_{H0}(G) < u_{HR}(G), \quad u_{00}(G) < u_{0R}(G).$$

Constraints on the rainfall level R

According to the formula for γ:

$$\gamma = \frac{\alpha}{c}\left[1 - z + mcR - \frac{R[c - \alpha(b + 3cn)]}{(1 - 2\alpha)(1 - \beta + \alpha\beta)}\right]$$

the constraints

$$0 \leq \gamma \leq \frac{R}{2}$$

imply that R should be not too small and not too large...

Other properties

...

We have:

- when $\gamma > 0$, and R small enough, the total ranking of controls hold:

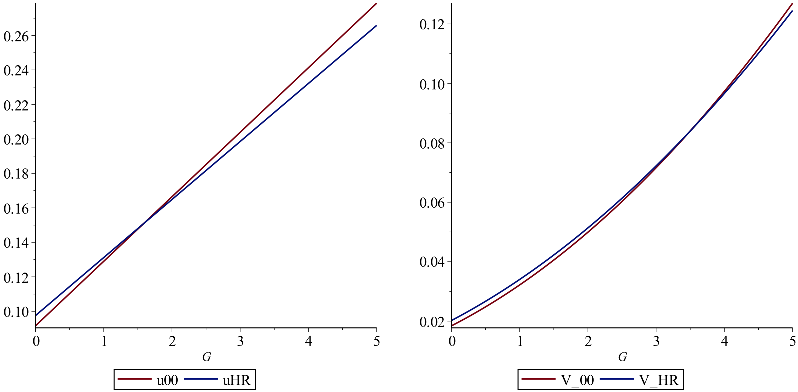
$$u_{H0}(G) < u_{HR}(G) < u_{00}(G) < u_{0R}(G).$$

- when $\beta \sim 0$ and $z < 1$, then $\gamma > 0$: the equilibrium control $u(G)$ is valid for all G
- when $\beta \sim 1$, $\gamma < 0$.

More analysis is needed for β large and R small.

When determining cost after harvesting and rainfall is a good economic and environmental option

Assume: $\beta = 0.3, b = 1, c = 0.04, z = 0.9, R = 1.$



Scarce resources, ctd

We have:

- $G_{00}^{\infty} < G_{HR}^{\infty}$, as always;
- $u_{00}(G) > u_{HR}(G), V_{00}(G) > V_{HR}(G)$, when G big, same ranking as in the myopic case: conflict;
- $u_{00}(G) < u_{HR}(G), V_{00}(G) < V_{HR}(G)$, when G small, reversed ranking: win-win.

When the level of the groundwater is small, setting costs after harvesting and rainfall is better from the economic *and* environmental point of view than the standard literature case where the cost is announced before rain and harvesting, *even* for quite myopic agents.

Conclusions and extensions

Conclusions:

- We illustrate the interest to charge users in function of their behavior, not just in function of the level of resource
- Possibility of win-win situations
- More analysis to better explain the phenomenon

Extensions:

- Stochastic case
- Stackelberg game with the regulator announcing the cost
- ...

Stochastic extension

Assume the recharge is a i.i.d. sequence $\{R_t; t = 0, 1, \ldots\}$.
In the LQ case, the Bellman equation becomes:

$$\begin{aligned} V^i(G_0) &= \max_{u_t^i} \Big[F_i(u^i) - C_i(G + m\mathbb{E}R - n(u_t^1 + u_t^2))u^i \\ &\qquad + \beta \mathbb{E} V^i(G_t + R - (u_t^1 + u_t^2)) \Big] \\ &= \max_{u_t^i} \Big[F_i(u^i) - C_i(G + m\mathbb{E}R - n(u_t^1 + u_t^2))u^i \\ &\qquad + \beta V^i(G_t + \mathbb{E}R - (u_t^1 + u_t^2)) + \frac{1}{2}\sigma_R^2 (V^i)'' \Big] \end{aligned}$$

and has the same controls as solution.